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5

Q - Meixner's Polynomial and Continued Fractions

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Abstract

In this paper an attempt has been made to establish continued fraction for the ratio of two q-meixner's polynomial using some known transformation formula in basic hyper geometric series.

Keywords and phrases: q-meixner's polynomial, basic hyper geometric series, and continued fractions.

1) Notations and known results

The basic hyper geometric series is defined as

$$ {}_r\phi_s\left[\begin{matrix}(a);\ z\\(b)\end{matrix}\right] = \sum_{n=0}^{\infty}\frac{((a);q)_n\, z^n}{((b);q)_n\,(q;q)_n}, \qquad (1.1)$$

where for the convergence of the series, we have $0<|q|<1$ and $|z|<1$ if r = s+1, and for any z if $r\le s$.

$$((a);q)_n = (a_1;q)_n (a_2;q)_n \ldots (a_r;q)_n \ and$$
$$((b);q)_n = (b_1;q)_n (b_2;q)_n \ldots (b_s;q)_n. \qquad (1.2)$$

The q-shifted factorial is defined as

$(a;q)_0 = 1$ and $(a;q)_n = (1-q)(1-aq)\ldots(1-aq^{n-1})$, n=1,2,3… (1.3)

The q-meixner's polynomial is defined as

$$M_n(x;\beta,\gamma,q) = (\beta;q)_n\, {}_2\phi_1\left[\begin{matrix}q^{-n}, x; q, \dfrac{q^{n+1}}{\gamma}\\ \beta;\end{matrix}\right]. \qquad (1.4)$$

[3; p.237 (23)]

We shall also use the following transformation formula in our analysis:

$$ {}_2\phi_1\left(q^{-n},b;c;q,z\right)=\frac{\left(\frac{c}{b};q\right)_n}{(c;q)_n}\,{}_3\phi_2\left(\begin{matrix}q^{-n},b,bzq^{-n};q,q\\0,bq^{1-n}/c;\end{matrix}\right) \tag{1.5}$$

[2; App.III (III.7)]

2) Main Results

In this paper we have established the following main results

$$ \text{(i)}\ \frac{M_n(x;\beta,\gamma,q)}{M_n(xq;\beta,\gamma,q)}=1-\frac{A_0}{\delta_x-}\frac{B_0}{1-}\frac{A_1}{\delta_x-}\frac{B_1}{1-....}, \tag{2.1}$$

where $A_i=\dfrac{xq^{1+n+i}\left(1-q^{-n+i}\right)}{\left(1-\beta q^{n+i}\right)\gamma}$, $B_i=\dfrac{\left(1-xq^{i+1}\right)\left(1-\frac{xq^{i+2}}{\gamma\beta}\right)q^{-n+1+i}}{\left(1-\frac{xq^{-n+i+2}}{\beta}\right)}$,

And $\delta_x=\dfrac{\left(1-\frac{\beta}{qx}\right)}{\left(1-\frac{\beta q^{n-1}}{x}\right)}$.

$$ \text{(ii)}\ \frac{(\beta;q)_n\,{}_2\phi_1\left(q^{-n},x;q,\frac{q^{n+1}}{\gamma}\right)}{(\beta;q)_n\,{}_2\phi_1\left(q^{-n+1},x;q,\frac{q^{n+1}}{\gamma}\right)}=1-\frac{C_0}{1-}\frac{D_0}{1-}\frac{C_1}{1-}\frac{D_1}{1-....}, \tag{2.2}$$

where for $i\geq 0$

$$ C_i=\frac{\left(1-xq^i\right)q^{i+1}}{\gamma\left(1-\beta q^{n+i}\right)},\ D_i=\frac{xq^i\left(1-xq^{-n+i+1}\right)\left(1-xq^{2+i}\right)}{\left(1-xq^{-n+i+2}\right)}. \tag{2.3}$$

Proof of (i): For $i\geq 0$,

$$ \text{Let } H_i=\left(\beta q^i;q\right)_n\,{}_2\phi_1\left(\begin{matrix}q^{-n+i},xq^i;q,\frac{q^{1+n}}{\gamma}\\\beta q^i;\end{matrix}\right), \tag{2.4}$$

and

$$ F_i=\left(\beta q^i;q\right)_n\,{}_2\phi_1\left(\begin{matrix}q^{-n+i},xq^{i+1};q,\frac{q^{1+n}}{\gamma}\\\beta q^i;\end{matrix}\right), \tag{2.5}$$

So,

$$F_i - H_i = (\beta q^i;q)_n xq^i \sum_{r\geq 1} \frac{(q^{-n+i};q)_r (xq^{i+1};q)_{r-1}}{(q;q)_{r-1}(\beta q^i;q)_r}\left(\frac{q^{1+n}}{\gamma}\right)^r, \tag{2.6}$$

$$= (\beta q^i;q)_n xq^i \sum_{r\geq 0} \frac{(q^{-n+i};q)_{r+1} (xq^{i+1};q)_r}{(q;q)_r(\beta q^i;q)_{r+1}}\left(\frac{q^{1+n}}{\gamma}\right)^{r+1},$$

$$= \frac{(1-q^{-n+i})xq^{n+i+1}}{(1-\beta q^{n+i})\gamma}(\beta q^i;q)_n \sum_{r\geq 0} \frac{(q^{-n+i+1};q)_r (xq^{i+1};q)_r}{(q;q)_r(\beta q^{i+1};q)_r}\left(\frac{q^{1+n}}{\gamma}\right)^r,$$

$$= \frac{(1-q^{-n+i})xq^{n+i+1}}{(1-\beta q^{n+i})\gamma}(\beta q^{i+1};q)_n \sum_{r\geq 0} \frac{(q^{-n+i+1};q)_r (xq^{i+1};q)_r}{(q;q)_r(\beta q^{i+1};q)_r}\left(\frac{q^{1+n}}{\gamma}\right)^r,$$

$$= \frac{(1-q^{-n+i})xq^{n+i+1}}{(1-\beta q^{n+i})\gamma}(\beta q^{i+1};q)_n \, {}_2\phi_1\left(\begin{matrix} q^{-n+i}, xq^{i+1}; q, q^{1+n}/\gamma \\ \beta q^{i+1}; \end{matrix}\right),$$

$= A_i H_{i+1}$, where A_i is given in (i).

So, $F_i - H_i =$. (2.7)

This gives
$$\frac{H_i}{F_i} = 1 - \frac{A_i}{F_i / H_{i+1}}. \tag{2.8}$$

Let us transform and using the transformation formula (1.5), we get:

$$F_i = \left(\beta/xq;q\right)_n \, {}_3\phi_2\left(\begin{matrix} q^{-n+i}, xq^{i+1}, xq^{i+2}/\gamma\beta; q, q \\ 0, xq^{-n+i+2}/\beta \end{matrix}\right), \tag{2.9}$$

And

$$H_i = \left(\beta/x;q\right)_n \, {}_3\phi_2\left(\begin{matrix} q^{-n+i}, xq^i, xq^{i+1}/\gamma\beta; q, q \\ 0, xq^{-n+i+1}/\beta \end{matrix}\right), \tag{2.10}$$

So,

$$\frac{\left(1-\beta/xq\right)}{\left(1-\beta q^{n-1}/x\right)} H_{i+1} - F_i = \left(\beta/xq;q\right)_n q^{-n+i} \sum_{r\geq 0} \frac{(q^{-n+i+1};q)_r (xq^{i+1};q)_r \left(xq^{i+2}/\gamma\beta;q\right)_{r+1}}{(q;q)_r \left(xq^{-n+i+2}/\beta q\right)_{r+1}} q^{r+1},$$

$$= \frac{(1-xq^{i+1})\left(1-xq^{i+2}/\gamma\beta\right)q^{-n+i+1}}{\left(1-xq^{-n+i+2}/\beta\right)}\left(\beta/xq;q\right)_n \sum_{r\geq 0} \frac{(q^{-n+i+1};q)_r (xq^{i+2};q)_r \left(xq^{i+3}/\gamma\beta;q\right)_r}{(q;q)_r \left(xq^{-n+i+3}/\beta;q\right)_r} q^r,$$

$$= \frac{\left(1-xq^{i+1}\right)\left(1-\frac{xq^{i+2}}{\gamma\beta}\right)q^{-n+i+1}}{\left(1-\frac{xq^{-n+i+2}}{\beta}\right)}\left(\frac{\beta}{xq};q\right)_n {}_3\phi_2\left(\begin{matrix} q^{-n+i+1}, xq^{i+2}, \frac{xq^{i+3}}{\gamma\beta}; q, q \\ 0, \frac{xq^{-n+i+3}}{\beta} \end{matrix}\right),$$

So, we have

$$\frac{\left(1-\frac{\beta}{xq}\right)}{\left(1-\frac{\beta q^{n-1}}{x}\right)} H_{i+1} - F_i = B_i F_{i+1}. \tag{2.11}$$

This gives

$$\frac{F_i}{H_{i+1}} = \delta_x - \frac{B_i}{\frac{H_{i+1}}{F_{i+1}}}. \tag{2.12}$$

Where and are given in (I).

Using (2.9) and (2.10) repeatedly and setting, we get (i).

Proof of (ii): For, $i \geq 0$

$$\text{let } H_i = \left(\beta q^i;q\right)_n {}_2\phi_1\left(\begin{matrix} q^{-n+i}, xq^i; q, q^{1+n}/\gamma \\ \beta q^i; \end{matrix}\right). \tag{2.13}$$

and

$$F_i = \left(\beta q^i;q\right)_n {}_2\phi_1\left(\begin{matrix} q^{-n+i+1}, xq^{i+1}; q, q^{1+n}/\gamma \\ \beta q^i; \end{matrix}\right). \tag{2.14}$$

So,

$$F_i - H_i = \frac{q^{i+1}\left(1-xq^i\right)}{\gamma\left(1-\beta q^{i+n}\right)}\left(\beta q^{i+1};q\right)_n \sum_{r\geq 0} \frac{\left(q^{-n+i+1};q\right)_r \left(xq^{i+1};q\right)_r}{(q;q)_r \left(\beta q^{i+1};q\right)_r}\left(\frac{q^{1+n}}{\gamma}\right)^r, \tag{2.15}$$

$$= C_i H_{i+1}, \text{ Where } C_i \text{ is given in (ii)}. \tag{2.16}$$

This gives

$$\frac{H_i}{F_i} = 1 - \frac{C_i}{\frac{F_i}{H_{i+1}}}, \tag{2.17}$$

Where A_i is given in (ii). (2.16)

Let transform using the transformation formula (1.5) we have;

$$F_i = \left(\beta/x;q\right)_n {}_3\phi_2\left(\begin{matrix} q^{-n+i+1}, xq^i, xq^{i+2}/\gamma\beta ; q, q \\ 0, xq^{-n+i+2}/\beta \end{matrix}\right), \tag{2.18}$$

and

$$H_i = \left(\beta/x;q\right)_n {}_3\phi_2\left(\begin{matrix} q^{-n+i}, xq^i, xq^{i+1}/\gamma\beta ; q, q \\ 0, xq^{-n+i+1}/\beta \end{matrix}\right), \tag{2.19}$$

So,

$$H_{i+1} - F_i = \frac{\left(1-q^{-n+i+1}\right)\left(1-xq^{i+2}/\beta\gamma\right)xq^{i+1}}{\left(1-xq^{-n+i+2}/\beta\right)}\left(\beta/x;q\right)_n \sum_{r\geq 0} \frac{\left(q^{-n+i+1};q\right)_r \left(xq^{i+1};q\right)_r \left(xq^{i+3}/\beta\gamma;q\right)_r}{(q;q)_r\left(xq^{-n+i+3}/\beta;q\right)_r} q^r,$$

$$= \frac{\left(1-q^{-n+i+1}\right)\left(1-xq^{i+2}/\beta\gamma\right)xq^{i+1}}{\left(1-xq^{-n+i+2}/\beta\right)} F_{i+1},$$

This gives

$$\frac{F_i}{H_{i+1}} = 1 - \frac{D_i}{H_{i+1}/F_{i+1}}, \tag{2.20}$$

Where D_i is given in (ii).

Now using (2.13) and (2.14) repeatedly and setting we get (ii).

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