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Dedicated to Prof. K. Srinivasa Rao

(75th Birth Anniversary)



$$\sum_{n=0}^{\infty} \frac{q^{n(n-1)}}{[q;q]_n} = \frac{1}{\left[q^2, q^3; q^5\right]_{\infty}} + \frac{1}{\left[q, q^4; q^5\right]_{\infty}}$$

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ON CERTAIN TRANSFORMATIONS OF BASIC HYPERGEOMETRIC FUNCTIONS USING BAILEY'S TRANSFORM

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Dedicated to Prof. K. Srinivasa Rao on his 75th Birth Anniversary

Abstract: In this paper, making use of Bailey transform and certain known summation formulas, we have established certain interesting transformation formulas of basic hypergeometric series.

Keywords and Phrases: Basic hypergeometric series, Bailey's pair and Bailey's transformation.

2010 Mathematics Subject Classification: 33D15

1. Introduction

The generalized basic hyper geometric series $_{r}\phi_{s}$ is defined by

$${}_{r}\phi_{s}\left[\begin{array}{c}a_{1},a_{2},\ldots,a_{r}\\b_{1},b_{2},\ldots,b_{s}\end{array};q,z\right]=\sum_{n=0}^{\infty}\frac{(a_{1},a_{2},\ldots,a_{n})_{n}}{(q,b_{1},b_{2},\ldots,b_{n})_{n}}[(-1)^{n}q^{n(n-1)/2}]^{1+s-r}z^{n} \quad (1.1)$$

where r and s are positive integers and |q| < 1. The above series converges absolutely for all z if $r \le s$ and for |z| < 1 if r = s + 1.

For real or complex a, q < 1, the q-shifted factorial is defined by

$$(a,q)_n = \begin{cases} 1 & \text{if } n = 0; \\ (1-a)(1-aq)(1-aq^2)\dots, (1-aq^{n-1}) & \text{if } n \in N. \end{cases}$$
 (1.2)

In 1947, Bailey established a remarkably simple and useful transformation formula which is given in the following form:

If

$$\beta_n = \sum_{r=0}^n \alpha_r u_{n-r} v_{n+r} \tag{1.3}$$

and

$$\gamma_n = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{r+2n} \tag{1.4}$$

where α_r, δ_r, u_r and v_r are functions of r only such that the series of γ_n exists, then under suitable conditions of convergence we have the following equation.

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.5}$$

In this paper, we shall use the following results due to Verma and Jain [10].

$${}_{4}\Phi_{3} \begin{bmatrix} a, q\sqrt{a}, -q\sqrt{a}, q^{-n}; -q^{-1/2+n} \\ \sqrt{a}, -\sqrt{a}, aq^{1+n} \end{bmatrix}$$

$$= \frac{(aq; q)_{n}}{2} \left[\frac{(-q^{-1/2}; q)_{n}}{(\sqrt{aq}; q)_{n}(-q\sqrt{a}; q)_{n-1}} + \frac{(-q^{-1/2}; q)_{n}}{(-\sqrt{aq}; q)_{n}(q\sqrt{a}; q)_{n-1}} \right]$$
(1.6)

$${}_{3}\Phi_{2}\left[\begin{array}{c}a,q\sqrt{a},q^{-n};-q^{n}\\\sqrt{a},aq^{1+n}\end{array}\right]=\frac{(aq,-1;q)_{n}}{2}\left[\frac{(1+\sqrt{a})}{(aq;q^{2})_{n}}+\frac{(1-\sqrt{a})}{(\sqrt{a};q)_{n}(-q\sqrt{a};q)_{n}}\right]$$
(1.7)

$${}_{2}\Phi_{1}\begin{bmatrix} a, q^{-n}; -q^{1/2+n} \\ aq^{1+n} \end{bmatrix}$$

$$= \frac{(aq, -\sqrt{q}; q)_{n}}{2} \left[\frac{(1+\sqrt{a})}{(-\sqrt{aq}; q)_{n}(q\sqrt{a}; q)_{n}} + \frac{(1-\sqrt{a})}{(\sqrt{aq}; q)_{n}(-q\sqrt{a}; q)_{n}} \right]$$
(1.8)

2. Main Results

In this section we shall establish the following results.

$$_{5}\Phi_{5}\left[egin{array}{c} a,q\sqrt{a},-q\sqrt{a},b,c;q;rac{-a\sqrt{q}}{bc} \ \sqrt{a},-\sqrt{a},qa/b,qa/c,0 \end{array}
ight] =rac{1}{2}\prod\left[egin{array}{c} aq,aq/bc;q \ aq/b,aq/c \end{array}
ight] imes$$

Proof of (2.1):

Let us choose $u_r = \frac{1}{(q;q)_r}, v_r = \frac{1}{(aq;q)_r}, \ \alpha_r = \frac{(a,q\sqrt{a},-q\sqrt{a};q)_r q^{r(r-1)/2}}{(q,\sqrt{a},-\sqrt{a};\overline{q})_r q^{r/2}},$ and $\delta_r = (b,c;q)_r (\frac{aq}{bc})^r$

Now using these in equations (1.2), (1.3) and using (1.6) we get the following:

$$\beta_n = \frac{(-q^{-1/2}; q)_n}{2} \left\{ \frac{1 + q^n \sqrt{a}}{(q, \sqrt{aq}, -q\sqrt{a}; q)_n} + \frac{1 - q^n \sqrt{a}}{(q, -\sqrt{aq}, q\sqrt{a}; q)_n} \right\}$$
(2.5)

and

$$\gamma_n = \frac{(b.c;q)_n}{(aq;q)_n} \left(\frac{aq}{bc}\right)^n {}_2\phi_1 \left[\begin{array}{c} bq^n, cq^n; q; \frac{aq}{bc} \\ aq^{1+2n} \end{array}\right]$$
(2.6)

Summing the series $_2\phi_1$, we have:

$$\gamma_n = \frac{(b.c;q)_n}{(aq/b,aq/c;q)_n} \left(\frac{aq}{bc}\right)^n \prod \begin{bmatrix} aq/b,aq/c;q\\ aq,aq/bc \end{bmatrix}$$
(2.7)

Putting these values in equation (1.5), we get the proof of (2.1).

Proof of (2.2):

Let us choose
$$u_r = \frac{1}{(q;q)_r}, v_r = \frac{1}{(aq;q)_r}, \ \alpha_r = \frac{(a,q\sqrt{a};q)_r q^{r(r-1)/2}}{(q,\sqrt{a};q)_r},$$

and $\delta_r = (b, c; q)_r (\frac{aq}{bc})^r$ Now using these in equations (1.2), (1.3) and using (1.7) we get the following:

$$\beta_{n} = \frac{1}{(q, aq; q)_{n}} {}_{3}\Phi_{2} \left[\begin{array}{c} a, q\sqrt{a}, q^{-n}; -q^{n} \\ \sqrt{a}, aq^{1+n} \end{array} \right]$$

$$= \frac{(-1; q)_{n}}{2(q; q)_{n}} \left\{ \frac{1 + \sqrt{a}}{(aq; q^{2})_{n}} + \frac{1 - \sqrt{a}}{(\sqrt{a}, -q\sqrt{a}; q)_{n}} \right\}$$
(2.8)

and

$$\gamma_n = \frac{(b, c; q)_n}{(aq; q)_n} \left(\frac{aq}{bc}\right)^n {}_2\phi_1 \left[\begin{array}{c} bq^n, cq^n; q; \frac{aq}{bc} \\ aq^{1+2n} \end{array}\right]$$
 (2.9)

Summing the series $_2\phi_1$, we have:

$$\gamma_n = \frac{(b, c; q)_n}{(aq/b, aq/c; q)_n} \left(\frac{aq}{bc}\right)^n \prod \begin{bmatrix} aq/b, aq/c; q \\ aq, aq/bc \end{bmatrix}$$
(2.10)

Putting these values in equation (1.5), we get the proof of (2.2)

Proof of (2.3):

Let us choose
$$u_r = \frac{1}{(q;q)_r}$$
, $v_r = \frac{1}{(aq;q)_r}$, $\alpha_r = \frac{(a;q)_r q^{r^2/2}}{(q;q)_r}$, and $\delta_r = (b,c;q)_r (\frac{aq}{bc})^r$

Now using these in equations (1.3), (1.4) and using (1.8) we get the following:

$$\beta_n = \frac{(-\sqrt{q}; q)_n}{2(q; q)_n} \left\{ \frac{1 + \sqrt{a}}{(-\sqrt{aq}, q\sqrt{a}; q)_n} + \frac{1 - \sqrt{a}}{(\sqrt{aq}, -q\sqrt{a}; q)_n} \right\}$$
(2.11)

and

$$\gamma_n = \frac{(b, c; q)_n}{(aq; q)_{2n}} \left(\frac{aq}{bc}\right)^n {}_{2}\phi_1 \left[\begin{array}{c} bq^n, cq^n; q; \frac{aq}{bc} \\ aq^{1+2n} \end{array}\right]$$
(2.12)

Summing the series $_2\phi_1$, we have:

$$\gamma_n = \frac{(b, c; q)_n}{(aq/b, aq/c; q)_n} \left(\frac{aq}{bc}\right)^n \prod \begin{bmatrix} aq/b, aq/c; q \\ aq, aq/bc \end{bmatrix}$$
(2.13)

Putting these in equation (1.5), we get the proof of (2.3)

Proof of (2.4):

In order to prove (2.4) we shall use the following summation formulas:

$${}_{6}\phi_{5}\left[\begin{array}{c} a,q\sqrt{a},-q\sqrt{a},b,c,q^{-n};q;\frac{aq^{1+n}}{bc}\\ \sqrt{a},-\sqrt{a},qa/b,qa/c,aq^{1+n} \end{array}\right] = \frac{(aq,aq/bc;q)_{n}}{(aq/b,aq/c;q)_{n}}$$
(2.14)

$${}_{2}\phi_{1}\left[\begin{array}{c}a,b;q;\frac{c}{ab}\\c\end{array}\right] = \frac{(c/a,c/b;q)_{\infty}}{(c,c/ab;q)_{\infty}}\tag{2.15}$$

Let us choose $u_r = \frac{1}{(q;q)_r}, v_r = \frac{1}{(aq;q)_r}, \ \alpha_r = \frac{(a,q\sqrt{a},-q\sqrt{a},b,c;q)_rq^{r(r+1)/2}}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c;q)_r} \left(\frac{-a}{bc}\right)^r,$ and $\delta_r = (d,e;q)_r(\frac{aq}{bc})^r$

Now using these in equations (1.3) and (1.4) we get the following:

$$\beta_n = \frac{1}{(q, aq; q)_n} \epsilon \phi_5 \left[\begin{array}{c} a, q\sqrt{a}, -q\sqrt{a}, b, c, q^{-n}; q; \frac{-aq^{1+n}}{bc} \\ \sqrt{a}, -\sqrt{a}, qa/b, qa/c, aq^{1+n} \end{array} \right]$$
(2.16)

Now summing the series $_6\phi_5$ using equation (2.14), we get:

$$\beta_n = \frac{(aq/bc;q)_n}{(q,aq/b,aq/c;q)_n} \tag{2.17}$$

and

$$\gamma_n = \frac{(d, e; q)_n}{(aq; q)_{2n}} \left(\frac{aq}{de}\right)^n {}_2\phi_1 \left[\begin{array}{c} dq^n, eq^n; q; \frac{aq}{de} \\ aq^{1+2n} \end{array}\right]$$
(2.18)

Summing the series $_2\phi_1$ using equation (2.15) we have:

$$\gamma_n = \frac{(d, e; q)_n}{(aq/d, aq/e; q)_n} \frac{(aq/d, aq/e; q)_\infty}{(aq, aq/de; q)_\infty} \left(\frac{aq}{de}\right)^n$$
(2.19)

Putting these values in equation (1.5), we get the proof of (2.4)

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